Admission to Ph.D. Program in Mathematics

Topics for Entrance Examination

Matrix Algebra: Rank of a Matrix, Linear dependence. Solutions of Linear Systems: Existence and Uniqueness. Eigen Values, Eigen Vectors, Properties of Eigen Values and Eigen Vectors, Cayley-Hamilton Theorem. Diagonalization of a Matrix, Power of a Matrix, Diagonalization by Orthogonal Transformation, Quadratic Forms.

Modern Algebra: Definition of Groups, Subgroups and Factor Groups, Lagrange's Theorem, Homomorphisms, Normal Subgroups. Quotients of Groups. Basic Examples of Groups including Symmetric Groups, Matrix Groups.

Real Analysis: Limit of Functions. Continuous Functions, Continuity and Compactness, Continuity and Connectedness, Discontinuities, Monotonic Functions, Infinite Limits and Limit at Infinity. Derivative of a Real Function. Mean Value Theorem, Continuity of Derivatives, L'Hospital's Rule, Derivatives of Higher Order, Taylor's Theorem.

Complex Analysis: Analytic functions, conformal mappings, bilinear transformations; complex integration: Cauchy's integral theorem. Zeros and singularities; Taylor and Laurent's series; residue theorem.

Vector Analysis: Divergence and curl of a vector point function – solenoidal and irrational functions – physical interpretation of divergence and curl of a vector point function. Integration of vector functions – Line, surface and volume integrals. Guass - Divergence Theorem – Green's Theorem – Stoke's Theorem.

Differential Equations:
ODE: General solution of homogeneous equations, non-homogeneous equations, Wronskian, method of variation of parameters.
PDE: Linear and quasilinear first order partial differential equations, method of characteristics; second order linear equations in two variables and their classification

Numerical Methods: Bisection method, fixed-point iteration, Newton's method. Error analysis for Iterative Methods. Computing roots of polynomials. Interpolation: Lagrange Polynomial. Divided Differences. Numerical differentiation; numerical integration: Trapezoidal and Simpson rules; numerical solution of systems of linear equations: direct methods (Gauss elimination, LU decomposition); iterative methods (Jacobi and Gauss-Seidel); numerical solution of ordinary differential equations: initial value problems: Euler's method, Runge-Kutta methods of order 2.

Statistics and Probability Theory: Probability, conditional probability, independent events, total probability and Baye's theorem. Random Variable, Probability density function, distribution function, mathematical expectation, variance, Discrete Distribution − Binomial, Poisson, Continuous Distribution − Normal distribution.